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LETTER TO THE EDITORS

COMMENT ON "ON THE TIME DEPENDENT CONVECTIVE HEAT TRANSFER IN FLUIDS WITH VANISHING PRANDTL NUMBER"

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NOMENCLATURE

m, exponent in power law free stream velocity variation;
q_1, heat flux;
r, defined in equation (6);
t, dimensionless time;
U_p, local non-dimensional free stream velocity;
x, dimensionless space coordinate along body.

Greek symbols
ψ, stream function;
τ, $= U_p t/x$ another dimensionless time.

Subscripts
w, conditions at wall;
ss, steady state.

Soliman and Chambre [1] have artfully used a combination of Fourier transforms and the method of characteristics to effect a solution to an important class of transient forced convection problems. However, this writer has detected an error in their work due, most likely, to a small oversight but which, unfortunately, eventually causes large errors in the response functions for the wall heat flux ratios and wall temperature ratios. A subsequent paper by Biasi [2], solving the same problem as [1] by approximate integral methods also contains the same mistake. The purpose of this note is to point out the nature of the error and rectify it and the various response functions given in [1] and [2].

The basic partial differential equation being solved in [1] is the following von Mises form of the transient thermal energy equation for an ultra-low Prandtl number fluid.

$$\frac{\partial T}{\partial t}+U_p(x)\frac{\partial T}{\partial x}=[U_p(x)]^2\frac{\partial^2 T}{\partial \psi^2}. \tag{1}$$

Employing the Fourier transform, equation (1) becomes a first order equation to which the method of characteristics is applied to yield the family of characteristic curves in the t–x plane, namely equation (23) of [1].

$$t-X(x)=C_1 \tag{23}$$

$$X(x)=\int_0^x \frac{\mathrm{d}\eta}{U_p(\eta)}. \tag{24}$$

The dividing characteristic, which joins the two different functional forms of the solution needed to satisfy the two side conditions, one on x and one on t, is given in [1] as, with $C_1 = 0$ in equation (23),

$$t = X(x) \quad \text{or} \quad x = F(t). \tag{27}$$

Of course we have that,

$$F(t)=X^{-1}(t) \tag{2}$$

where X^{-1} is the inverse of the function X in equation (24).

Next, looking at equations (23) and (27), Soliman and Chambre claim that equation (23) can be re-written as

$$x-F(t)=C_3. \tag{28}$$

This, however, is not possible unless either $X(x)$ is a linear function of x (such as is the case for flow with zero pressure gradient over a surface), or unless $C_1 = 0 = C_3$ which is the dividing characteristic. This is clearly seen if one proceeds more formally from equation (23) as shown below.

$$X(x)=t-C_1. \tag{3}$$

Hence

$$x=X^{-1}[t-C_1]. \tag{4}$$

This result does not give equation (28) unless one of the two conditions just mentioned are satisfied. Using the correct result, equation (4) of this note, in place of equation (28) of [1], this writer has found that to rectify equations (30), (34), (36), and (39) in [1], one must replace $x-F(t)$ wherever it appears in these equations by $X^{-1}[X(x)-t]$ or, equivalently, by $F[X(x)-t]$.

The statement, in [1], that their equation (34) satisfied the partial differential equation was checked for the case of an abrupt change in wall temperature. It was found that (34) satisfied the partial differential equation only along the dividing characteristic or only for the case where $m = 0$, consequences that are expected because of the discussion preceeding equation (3) of this note. On the other hand, equation (34) when corrected as shown above satisfied the differential equation for every x, t, ψ, and value of m in its solution domain restricted by $t \leq X(x)$.

Similarly, in Biasi's work [2], $x-F(t)$ must be replaced by $F[X(x)-t]$ wherever it appears. In addition, there seems to be an η left out of the integrand in both equations (27) and (28) of [2].* The missing η doesn't affect the numerical results because only the case where the integrals are zero is dealt with explicitly.

For the case of power law free stream velocity variations and a step change in the body surface temperature, combination of equations (45) and (47) of [1] yields, for the time varying wall flux ratio,

$$q_{1,w}/q_{1,w,ss}=\frac{1}{(1-\{1-[(1-m)\tau]^{1/(1-m)}\}^{m+1})^{1/2}}$$

whereas the corrected result is given as equation (5) below and also is valid for [2] as well.

$$q_{1,w}/q_{1,w,ss}=\frac{1}{\left\{1-\left[1-\dfrac{(1-m)\tau}{(1+m)}\right]^{(m+1)/(1-m)}\right\}^{1/2}}. \tag{5}$$

The step change in wall temperature solution of [1] is in error qualitatively, as well as quantitatively, since it predicts a wall flux ratio which increases as m increases at fixed non-dimensional time τ while, in fact, the ratio given by equation (5) decreases as m increases at fixed τ. Note that, as mentioned before, equation (5) and the corresponding equation of [1] are the same when $m = 0$ for here $X(x)$ is simply x thus allowing equation (4) of this note to reduce to equation (28) of [1].

For the thermal transient induced by a step change in the wall flux when the free stream velocity variation is of the power law type, equation (53) of [1] can be corrected by replacing their α, in the incomplete Beta function, by the following corrected parameter r.

$$r=[1-\tau(1-m)]^{(m+1)/(1-m)}. \tag{6}$$

*A referee has kindly pointed out that there is a δ^2 missing from the third term on the RHS of equation (14) of [2].

Table 1. Comparison between results of [1] and the corrected values from this note for $m = 0.5$

Non-dimensional time τ	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
$T_w/T_{w,ss}$ (from [1])	0.10	0.195	0.30	0.40	0.51	0.62	0.76	1.0
Corrected $T_w/T_{w,ss}$	0.46	0.64	0.76	0.85	0.92	0.96	0.99	1.0

To illustrate the differences between the wall temperature ratio response functions from equation (53) of [1] and the same equation corrected by use of equation (6), sample calculations were made with $m = 0.5$ and are given in Table 1. The entries from [1] came from their Fig. 4. In this note the incomplete Beta function was evaluated by graphical integration. As a measure of the accuracy of the integration, the writer compared his computed value of the complete Beta function with its equivalent in terms of gamma functions listed in [3] and agreement was within 0.5%.

Finally, the corrected version of equation (38) in [2] for the response to an abrupt change in the surface heat flux when the free stream velocity is a power law variation is as follows:

$$T_w/T_{w,ss} = \{1-[1-(1-m)\tau]^{1/(1-m)}\}^{1/2}, \quad \tau \leq \frac{1}{1-m}. \tag{7}$$

In conclusion, this writer enjoyed reading, studying, and using reference [1] and it is hoped that the corrections outlined here will be of aid to others who may use [1] and [2].

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